

ABSTRACTS OF ARTICLES DEPOSITED AT VINITI*

FREE-CONVECTION HEAT TRANSFER IN A CLOSED THREE-DIMENSIONAL CAVITY

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UDC 536.25

This paper gives the results of an investigation of free convection from a cylindrical heater centrally or eccentrically positioned in a cavity in the shape of a cylinder or parallelepiped. It is shown that the position of the heat source within the closed cavity has no effect on heat transfer so long as the boundary layers on the heat-transfer surfaces can develop freely. The heat transfer can be calculated as for a centrally situated heater. The results of investigations of free convection within a cylindrical or parallelepiped-shaped cavity with a central heater are correlated in the form of equations of the type $NU = f(Ra^n)$. It is characteristic that the exponent is a variable that depends on the Rayleigh number and is greater than 0.25 — the value obtained by using a boundary-layer model for the investigation of free convection.

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INVESTIGATION OF INTEGRAL EMISSIVITY OF NICKEL NEAR CURIE POINT

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The temperature dependence of the hemispherical integral emissivity ϵ of nickel was investigated by the modulation method [1]. The specimen is surrounded by a radiation shield and is placed in a vacuum chamber. Shield temperature oscillations around a mean value are imposed. The specimen is supplied with power, which oscillates around a mean value in counterphase with the shield temperature oscillations. At a certain value of the amplitude P_{osc} of the power oscillations the amplitude of the temperature oscillations of the specimen becomes zero. In this case the emissivity of the specimen can be determined from the equation

$$\epsilon = \frac{P_{osc}}{4\sigma T_s^3 \theta_s},$$

where σ is the Stefan-Boltzmann constant; S , surface area of the specimen; T_0 , mean temperature of the shield; θ_s , amplitude of the shield temperature oscillations.

The denominator in the equation depends on the shield heat regime (parameters T_s and θ_s), which can be kept constant for different values of T_0 , the mean temperature of the specimen. The nature of the temperature dependence of ϵ is determined in this case by the function $P_{osc} = P_{osc}(T_0)$. This ensures high sensitivity of the method to change in ϵ .

The measurements were made on the apparatus described in [1]. A more uniform temperature field in the specimen was obtained by indirect heating with an auxiliary heater.

Near the Curie point of the specimen an anomalous value of the amplitude of the heater power oscillations required to suppress the specimen temperature oscillations was found. This confirms the anomalous nature of the temperature dependence of ϵ in the region of the phase transition.

Numerical values of the emissivity ϵ of nickel at 300–450°C were obtained. The results of other investigations, particularly [2], where an anomalous value of ϵ was found for nickel and iron close to the Curie point, are cited and discussed.

LITERATURE CITED

1. V. Ya. Cherepanov, Inventor's Certificate No. 631788; Byull. Izobr., No. 41 (1978).

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SOLUTION OF PROBLEM OF FLOW STABILIZATION IN LAMINAR NATURAL CONVECTION

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A stationary stabilized flow in laminar natural convection denotes a hypothetical movement of the medium in which the flow parameters - velocities and temperatures - are independent of the coordinates measured along the direction of the flow and of the time. It follows from the definition that the term "stabilized flow" is identical with the term "uniform flow," i.e., motion without acceleration in the direction of propagation.

We consider a plane laminar flow of incompressible medium with constant thermophysical properties. In correspondence with the above definition the Navier-Stokes equations take the form

$$-\mu \frac{d^2u}{dy^2} + \rho\beta g(t - t_c) = 0; \quad -\lambda \frac{d^2t}{dy^2} + \mu \left(\frac{du}{dy}\right)^2 = 0.$$

The solution of this system is given by the following functions:

$$u = \frac{27}{2} \left(\frac{20}{9} \cdot \frac{\lambda}{\rho\beta g} \right)^{\frac{1}{3}} / \left[\left(\frac{9}{20} \cdot \frac{\rho\beta g}{\lambda} \right)^{\frac{1}{3}} y + A_1 \right]^2 + A_2;$$

$$t = t_c + 81 \sqrt[3]{\frac{9}{\mu}} / \left[\sqrt[3]{\frac{20\lambda}{\rho\beta g}} \left(y \sqrt[3]{\frac{9}{20} \cdot \frac{\rho\beta g}{\lambda}} + A_1 \right)^4 \right]$$

where A_1 and A_2 are arbitrary constants.

These solutions reveal that stabilization of the flow in laminar natural convection occurs in cases where a heated body moves along with the medium under the action of the repulsive buoyancy force, and the fluid flows over the outside of the body. In other cases stabilization does not exist.

If μ is put equal to 0 in the above system of equations, the functions for u and t at $\mu = 0$ will satisfy the system for any boundary conditions. This means that a partial solution is obtained for the problem posed by O. A. Ladyzhenskaya, viz., the solution of the boundary-value problem for the Navier-Stokes equations may tend to the solution of the boundary-value problem for an ideal fluid when the viscosity μ tends to zero irrespective of the kind of boundary conditions.

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HEAT-CONDUCTION PROBLEM FOR A THREE-LAYERED PLATE

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The temperature field in a three-layered plate of asymmetric structure, whose side surfaces are thermally insulated, is found.

Initially the outer layers of the plate have temperature t_0 , and the middle layer t_1 . In this case the initial temperature of the plate, as a unit whole, can be put in the form

$$t|_{\tau=0} = t_0 + (t_1 - t_0) S_-(x - x_1) + (t_0 - t_1) S_-(x - x_2),$$

where $S_-(x)$ is an asymmetric unit function [1]; τ is the time.

The thermophysical characteristics of the system are assigned in a similar way.

To determine the temperature field in the plate we have the differential heat-conduction equation [2]

$$\frac{\partial}{\partial x} \left[\lambda(x) \frac{\partial t}{\partial x} \right] = c(x) \rho(x) \frac{\partial t}{\partial \tau},$$

where $\lambda(x)$ is the thermal conductivity; $c(x)$, specific heat; $\rho(x)$, density.

Substituting the expressions for the thermophysical characteristics, assigned with the aid of asymmetric unit functions, into the above equation and using the relation involving multiplication of asymmetric delta functions by the asymmetric unit functions, we obtain an equation with discontinuous coefficients:

$$\frac{\partial^2 t}{\partial x^2} = \left[\frac{1}{a_0} + \left(\frac{1}{a_1} - \frac{1}{a_0} \right) N(x) \right] \frac{\partial t}{\partial \tau} - (\lambda^* - 1) \delta_-(x - x_1) \frac{\partial t}{\partial x} \Big|_{x=x_1} + \left(1 - \frac{1}{\lambda^*} \right) \delta_-(x - x_2) \frac{\partial t}{\partial x} \Big|_{x=x_2}.$$

Here $\delta_-(x)$ is the Dirac asymmetric delta function [1, 2];

$$N(x) = S_-(x - x_1) - S_-(x - x_2); \lambda^* = \frac{\lambda_1}{\lambda_2}.$$

Quantities with the subscript 0 relate to the outer layers, and those with 1 to the middle layer.

Applying the Laplace time transformation to the obtained equations and then multiplying successively the right and left sides of this equation by $S_-(x - x_2)$ and $S_-(x - x_1)$, we obtain a system of two equations with constant coefficients for the new unknowns $U_1 = \bar{t} S_-(x - x_1)$ and $U_2 = \bar{t} S_-(x - x_2)$. Then, substituting the solutions for U_1 and U_2 into the initial equation we find the temperature image \bar{t} .

Conversion to the original is effected by the well-known Vashchenko-Zakharchenko expansion theorem.

The temperature distribution in the plate in relation to the coordinate (thickness) and time was also investigated. The results are given in the form of graphs.

LITERATURE CITED

1. G. A. Korn and T. A. Korn, *Mathematical Handbook for Scientists and Engineers*, McGraw-Hill, New York (1968).
2. Ya. S. Podstrigach and Yu. M. Kolyano, *Unsteady Temperature Fields and Stresses in Thin Plates [in Russian]*, Naukova Dumka, Kiev (1972).

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MATHEMATICAL MODEL OF COMPRESSION AND EXPANSION OF A VAPOR-GAS-LIQUID MIXTURE

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An analysis of the special features of compression and expansion of a vapor-gas-liquid mixture shows that these processes are accompanied by mass-transfer interaction between the vapor-gas (compressible) phase and the liquid (incompressible phase) due to condensation of the vapor (evaporation of liquid) and solution of the gas in the liquid. The composition of elements connected to the compressible phase or separated from it differs from the composition of the acting elements. The same feature is observed when the pressure of the mixture is altered solely by external heat transfer. The difference in the composition of the connected and separated elements from the composition of the acting elements leads to a change in composition and, hence, to a change in the thermodynamic properties of the compressible phase.

The aim of the work was to devise a mathematical treatment for the description of the compression and expansion of vapor-gas-liquid mixtures which takes into account the variation of the mass and the composition of the vapor-gas phase of the mixture.

This problem was solved by using the following assumptions: 1) the vapor-gas-liquid mixture is in heat- and mass-transfer equilibrium; 2) the vapor and gas conform to the ideal gas laws; 3) the liquid is incompressible; 4) the compression chamber is airtight. The solubility of the gas in the liquid was taken into account by the volume solubility coefficient, which is equal to the volume of gas dissolved in unit volume of liquid referred to the temperature and partial pressure of the gas.

Equations for the pressure and temperature in relation to volume and external heat transfer were obtained by using the laws of conservation of matter and energy.

If during the compression (expansion) the change in temperature is slight and the vapor density is low in comparison with the liquid density, the change in partial pressure of the gas p_G is given by the equation

$$p_G(V + \kappa V_L) = G_1(V_1 + \kappa V_{L1}) = \text{const},$$

where V is the volume of the vapor-gas phase; V_L , volume of the liquid phase; κ , coefficient of volume solubility of the gas in the liquid; 1, initial point of the process.

The structure of (1) is analogous to the structure of the equation $pV = \text{const}$, which represents the isothermic compression of a gas. A comparison of these equations indicates that in the presence of undissolved gas the liquid volume κV_L obeys the gas-compression laws.

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TWO-DIMENSIONAL TEMPERATURE WAVES IN A MEDIUM WITH A FINITE HEAT-TRANSFER RELAXATION PERIOD

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In the (x, y) plane at temperature $T_0 \equiv \text{const}$ there is a motionless medium in which heat-transfer processes are given by the relations

$$\begin{aligned} cT_t + c\gamma T_{tt} &= (\lambda T_x)_x + (\lambda T_y)_y, \quad \lambda = \lambda_0 \exp(\lambda T), \\ \gamma &= \text{const}, \quad c/\lambda = \kappa = \text{const}, \quad \gamma\kappa = n^2, \quad \omega_0 n = 1, \\ \theta(u, v, t) &= xu + yv - T(x, y, t) + Mt, \quad T_x = u, \quad T_y = v, \quad M = \text{const}, \\ x &= \theta_u, \quad y = \theta_v, \quad u = r \cos \varphi, \quad v = r \sin \varphi, \quad \partial(x, y)/\partial(r, \varphi) \neq 0, \\ x(r, \varphi, t) &= \theta_r \cos \varphi - r^{-1} \theta_\varphi \sin \varphi, \quad y(r, \varphi, t) = \theta_r \sin \varphi + r^{-1} \theta_\varphi \cos \varphi. \end{aligned}$$

Here T is the temperature; t , time; c , volumetric specific heat; λ , thermal conductivity of the medium; γ , heat-transfer relaxation period; θ , new required function obtained by a Legendre transformation of the initial equation. The equation obtained for $\theta(r, \varphi, t)$ is:

$$\begin{aligned} \theta_{rr} + r^{-1} \theta_r + r^{-2} \theta_{\varphi\varphi} + n^2 (2r^{-2} \theta_{\varphi r} \theta_{\varphi t} - r^{-2} \theta_{rr} \theta_{\varphi t}^2 - r^{-1} \theta_r \theta_{\varphi t}^2 - \\ - r^{-2} \theta_{\varphi\varphi} \theta_{rt}^2 - 2r^{-3} \theta_{\varphi r} \theta_{\varphi t}) + (n^2 \theta_{tt} + lr^2 - \kappa M + \kappa \theta_t) \delta^{-1} = 0, \\ \delta^{-1} = r^{-1} \theta_r \theta_{rr} + r^{-2} \theta_{rr} \theta_{\varphi\varphi} - r^{-4} \theta_\varphi^2 + 2r^{-3} \theta_{\varphi r} \theta_{r\varphi} - r^{-2} \theta_{r\varphi}^2, \end{aligned}$$

a solution is obtained by the method in [1] in the form of a convergent series,

$$\begin{aligned} \theta(r, \varphi, t) &= \sum_{n=0}^{\infty} a^{(n)}(\varphi, t) r^n, \quad a^{(0)} = -T_0 + Mt, \\ a^{(1)}(\varphi, t) &= \omega_0 t + f(\varphi), \quad a^{(2)}(\varphi, t) = C^{(2)}(\varphi) (t + \tau_0)^{\frac{1}{2}} \exp\left(\frac{\kappa t}{2n^2}\right), \quad \tau_0 = n(f + f''), \\ a^{(3)}(\varphi, t) &= (t + \tau_0) \tau_0^{-\frac{1}{2}} C^{(3)}(\varphi) \exp\left(\frac{\kappa t}{n^2}\right) + \frac{1}{3n} (t + \tau_0) C^{(2)}(\varphi) S \exp\left(\frac{\kappa t}{n^2}\right) + \\ &+ \frac{t + \tau_0}{6} \left[b_1 t + b_2 \left(\frac{1}{\tau_0} - \frac{1}{t + \tau_0}\right) + \frac{b_3}{2} \left(\frac{1}{\tau_0^2} - \frac{1}{(t + \tau_0)^2}\right) + \frac{b_4}{3} \left(\frac{1}{\tau_0^3} - \frac{1}{(t + \tau_0)^3}\right) \right] \exp\left(\frac{\kappa t}{n^2}\right). \end{aligned}$$

$$S(\varphi, t) = \int_0^t \frac{\exp(-\kappa t/2n^2)}{t + \tau_0} dt, \quad b_1 = \frac{\kappa^2}{2n^3} (C^{(2)})^2,$$

$$2b_2 = -11n(C^{(2)})^2 - \kappa(f' + f''')C^{(2)}C_\varphi^{(2)} + 2n(C_\varphi^{(2)})^2 - 4nC^{(2)}C_{\varphi\varphi}^{(2)};$$

$$-b_3 = 2n^2(f' + f''')C^{(2)}C_\varphi^{(2)} + n^2(f' + f^{IV})C^{(2)}^2, \quad 4b_4 = n^2(f' + f''')^2(C^{(2)})^2,$$

and a recurrent formula for $a^{(k+2)}(\varphi, t)$, $k \geq 0$ is found. The movable boundary of the region is assigned parametrically: $r = r_b(\beta, t)$, $\varphi = \varphi_b(\beta, t)$, the only restriction being that the conditions $t=0$: $r_b(\beta, 0) = 0$, $\varphi_b(\beta, 0) = \varphi$, $\beta = \varphi$ are satisfied; hence it is easy to obtain the law of motion of the boundary in the physical plane. The equation of the temperature wave front $r = 0$ is represented in the form

$$x_f(\varphi, t) = (\omega_0 t + f) \cos \varphi - f' \sin \varphi, \quad y_f(\varphi, t) = (\omega_0 t + f) \sin \varphi + f' \cos \varphi,$$

the boundary of the region and the wave front coincide at $t = 0$, and their initial form can be altered by assignment of $f(\varphi)$. At $t = 0$ the boundary temperature is constant and equal to T_0 , and subsequently varies according to the law

$$T_b(\beta, t) = T_0 + (x \cos \varphi + y \sin \varphi)_b r'_b - (\omega_0 t + f_b) r_b - \sum_{k=0}^{\infty} a^{(k+2)}(\varphi_b, t) r_b^{k+2},$$

which is determined within the framework of this class of solutions by a choice of the arbitrary function $C^{(k+2)}(\varphi)$, $k \geq 0$.

When $C^{(2)}(\varphi) \neq 0$, $n(f + f') > 0$ for this class of temperature fields the "gradient catastrophe" does not occur.

In the special case of $C^{(k+2)}(\varphi) \equiv \text{const}^{(k+2)}$, $f(\varphi) \equiv \text{const}$, $r_b = r_b(t)$ the obtained solution gives the nonstationary uniform temperature field with cylindrical symmetry, $T = T(r, t)$.

The literal subscript denotes partial differentiation: f is the value of the function on the wave front, b is the value on the boundary of the region.

LITERATURE CITED

1. A. F. Sidorov, Prikl. Mat. Mekh., 35, No. 3 (1971); Prikl. Mat. Mekh., 36, No. 3 (1972). Reg. No. 2100-79 Dep. Original article submitted January 8, 1979; abstract submitted April 24, 1979.

MECHANISM OF HEAT AND MASS TRANSFER IN ZONE OF CONTACT OF A HEAT-PRODUCING ELEMENT WITH SURFACE OF SUBLIMING SOLID CRYOAGENT

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Conductive supply of heat from a plane vapor-impermeable heat-producing element to a porous solid cryoagent produces a complex heat- and mass-transfer process in a zone of contact. As a first approximation we assume that the heat flux q_0 is transmitted to the solid cryoagent only through contact "spots" and leads to its sublimation from the free surface of the pores. The temperatures of the heat-producing element and the cryoagent on their contact surface are equal. Solving the heat-conduction equation for a porous substance with due regard to internal heat sinks due to sublimation [1], we obtain an equation representing the temperature distribution on the surface of the heat-producing element in the contact zone T_H :

$$T_H = T_0' + \frac{q_0}{\lambda_T(1-\Pi)\sqrt{A}}, \quad (1)$$

where

$$A = 3.704 \frac{S_0}{\lambda_T} \rho_T r \frac{r\mu}{RT_0} \exp\left(-\frac{r\mu}{RT_0}\right). \quad (2)$$

Owing to the hydraulic resistance of the pores to the vapor flow the pressure under the surface of the heat-producing element increases, and the flux passes not only through the zone

of direct contact, but occupies a layer of thickness δ . The sublimation temperature T_0' of the cryoagent comes into equilibrium with the local vapor pressure p .

The value of δ can be determined from the heat-balance equation for the layer of cryoagent through which the vapor flows, provided that this flow is laminar:

$$\delta = 0,273 \frac{d_e(1-\Pi)\sqrt{A}}{S_v} \cdot \frac{\lambda_T}{\lambda_v} \quad (3)$$

The distribution of vapor pressure under the surface of the heat-producing element is found from the solution of the D'Arcy equation [2], and for the case of outflow of vapor at subsonic velocity from the free surface of the cryoagent, has the form

$$p^2 = p_0^2 + \frac{q_0}{k_\phi} \frac{RT_0'}{r\mu} \left[\frac{R_H^2 - \rho^2}{2\delta} + \frac{R_H}{\sqrt{2}} \frac{K_0\left(\sqrt{2} \frac{R_H}{\delta}\right)}{K_1\left(\sqrt{2} \frac{R_H}{\delta}\right)} \right] \quad (4)$$

In the presence of a zone of sonic outflow the pressure distribution can be expressed as follows:

$$p^2 = \left(\frac{2p_0}{2+k} \right)^2 + \frac{q_0}{k_\phi} \frac{RT_0'}{r\mu} \frac{R_H^2 - \rho^2}{2\delta} + \frac{p_0 c (\alpha - 1) R_H^2}{2k_\phi \delta} + \left[\alpha R_H + \frac{\sqrt{2}}{4} \frac{k(k+2) k_\phi p_0}{c} \frac{K_1\left(\sqrt{2\alpha} \frac{R_H}{\delta}\right)}{K_0\left(\sqrt{2\alpha} \frac{R_H}{\delta}\right)} \right] \frac{p_0 c}{k_\phi \delta} \alpha R_H \ln \alpha \quad (5)$$

The coefficient α is determined from the equation of material balance of the fluxes on the free surface of the cryoagent

$$\frac{q_0}{p_0 c} \frac{RT_0'}{r\mu} + 1 - \alpha^2 = \alpha \frac{\sqrt{2} \delta}{R_H} \cdot \frac{K_1\left(\sqrt{2\alpha} \frac{R_H}{\delta}\right)}{K_0\left(\sqrt{2\alpha} \frac{R_H}{\delta}\right)} \quad (6)$$

The temperature dependence of the saturated vapor pressure can be used to determine T_0' and from Eq. (1) the temperature T_H of the surface of the heat-producing element.

A comparison of the theoretical and experimental data shows that the disagreement does not exceed 5%.

NOTATION

ρ , coordinate; R_H , radius of heat-producing element; p_0 , vapor pressure; d_e , Π , S_v , equivalent pore diameter, voidage, and specific surface, respectively, of porous cryoagent; r , μ , k , heat of sublimation, molecular weight, and adiabatic exponent; c , local sound velocity; R , universal gas constant; ρ_T , k_ϕ , density and permeability of porous cryoagent; λ_T , λ_v , thermal conductivities of cryoagent and vapor; $K_0(x)$, $K_1(x)$, Hankel functions.

LITERATURE CITED

1. B. Ya. Lyubov, Theory of Crystallization in Large Volumes [in Russian], Nauka, Moscow (1975).
2. A. V. Lykov, Heat and Mass Transfer (Handbook) [in Russian], Énergiya, Moscow (1978).

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